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Planning of capacity, production and inventory decisions in a generic reverse supply chain under uncertain demand and returns

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There is a growing interest for the design and operation of reverse supply chain systems due to the cost and the legislation issues. In this paper, we address the disassembly, refurbishing and production operations in a reverse supply chain setting for modular products such as computers and mobile phones considering the uncertainties in this system, which are the return amounts of the used products and demand for final products. We develop a large-scale mixed integer programming model in order to capture all characteristics of this system, and use two-stage stochastic optimisation and robust optimisation approaches to analyse the system behaviour. In the first stage, we focus on the strategic decisions about the capacities at disassembly and refurbishing sites considering different scenarios regarding the uncertainties in the system. In the second stage, we analyse the operational decisions such as production, inventory and disposal rates. We observe through our extensive numerical analysis that the randomness of demand and return values effect the performance of the system substantially and the uncertainty of the return amounts of used products is much more important than the uncertainty of demand in this system.

Keywords: capacity planning; reverse supply chain; re-manufacturing; stochastic optimisation; robust optimisation

1. Introduction and literature review

The main driver of the supply chains in the manufacturing industries has become the adaptation speed of innovation in the products. Therefore, the life span of these products shortened significantly compared to a decade ago. Manufacturing firms started to realise that used and returned products are also important due to valuable parts and materials in addition to new products, and society is very sensitive towards sustainability and environment. Moreover, some people might prefer recycled or remanufactured products. Jayaraman, Raymond, and Rolland (2003) imply that to increase profitability and survival, companies must coordinate their activities including environmental considerations in the reality of societal pressures. Besides, municipalities and governments encourage these operations to decrease the incineration activities and landfill areas. The research and corporate communities want to answer societal pressures for increasing sustainability, while minimising costs and perhaps increasing profitability.

In this paper, we address the disassembly, refurbishing and production operations in a closed loop supply chain setting for modular products such as computers and mobile phones considering the uncertainties in this system, which are the return amounts of the used products and demand for the final products. We model a generic disassembly and remanufacturing system and develop a large-scale mixed integer programming model in order to capture all characteristics of this system and use two-stage stochastic optimisation and robust optimisation approaches to analyse the system behaviour. In the first stage, we focus on the strategic decisions about the capacities at disassembly and refurbishing sites considering different scenarios regarding the uncertainties in the system. In the second stage, we analyse the operational decisions such as production, inventory and disposal rates.

Traditional supply chains are defined as a network of different business echelons such as vendors, manufacturers, distributors, controlling their processes together (Beamon 1998). The forward flow of raw materials, intermediate products and final products and the backward flow of information are managed efficiently. Practitioners try to decide on inventory levels, production quantities, the locations and number of echelons under certain or uncertain demand. However, the closed loop supply chain involves more activities than for traditional supply chains. For instance, Kumar and Malegeant (2006) define the closed loop supply chain as redesign of traditional supply chain; in addition to the forward flow of the raw materials and products, the backward flow of used and returned products are also considered. Dekker et al. (2004) give a detailed definition of closed loop supply chain: 'The process of planning, implementing and controlling backward flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal'.

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When we compare the definition of closed loop supply chain and traditional supply chain, we see that planning for closed loop supply chains involves more tasks than that for traditional supply chains. Difficulty of the closed loop supply chain management comes from not only the increased number of echelons or processes but also the complexity of the dynamics. Tibben-Lembke and Rogers (2002) summarise the differences between traditional supply chain and the closed loop supply chain. The most important closed loop supply chain characteristics which increase complexity of the supply chain are listed as:

- two types of uncertain variables which are returns and demand,
- two types of transportation operations which are distribution and collection,
- the quality and packagings are mostly not standardised, and
- the pricing of the returns and demand are uncertain.

Sarkis et al. (1995) explain the difficulties in reverse supply chain operations. Supply chain systems are originally designed for forward channels, reverse distribution costs are higher and returns may not be transported or handled as easy as first-hand products. The closed loop supply chain literature is focused to particular echelons of the complex network. Jayaraman, Guide, and Srivastava (1999) and Fleischmann et al. (2001) model closed loop supply chains as reverse logistics model and try to find the locations of facilities. They also detect that increasing the return rate decrease the total cost for facility location models. Demirel and Gokcen (2008) propose a mixed-integer mathematical model for network optimisation in integrated reverse logistics to find the optimal locations of disassembly, collection and distribution centers while determining the transportation and production quantities. Barros, Dekker, and Scholten (1998) study a two-level network design for a recycling sand case study with the aim of increasing the recycling construction waste to comply with government regulations. Jayaraman (2006) uses a deterministic mixed integer linear programming model for production planning in closed loop supply chains. Product recovery and reuse are the main operations in this model. Franke et al. (2006) study a generic model for remanufacturing of cellular phones.

The challenge of the closed loop supply chain activities are driven by uncertain demand from forward flows of supply chain and uncertain returns of backward flows of supply chain. Guide, Harrison, and Van Wassenhove (2008) mention that companies do not have much information related to the quality, quantity and timing of the returned products. To handle the uncertainty of demand and returned products, different approaches including stochastic optimisation (SOA) or robust optimisation (ROA) can be used.

In SOA, different scenarios are integrated into a large-scale mixed integer optimisation problem and all of these scenarios are solved to evaluate the same strategic decisions. In ROA, the deviation between optimum costs and expected cost are minimised while having the same values for strategic decisions in all scenarios.

The application of stochastic and robust optimisation for solving closed loop supply chains has recently started to emerge in the literature. Chouinard, D'Amours, and At-Kadi (2008) propose a stochastic optimisation model to reduce the impacts of randomness in demand, supply and processing. Listes (2007) explains the stochastic optimisation problems in network design that the location decisions are given in the first stage before they know the scenarios and then they give product flow decisions in the second stage. Multi-period multi-echelon network design for integrated logistics problem which is solved by stochastic optimisation approach is modeled by El-Sayed, Afia, and El-Kharbotly (2010). Realff, Ammons and Newton (2004) try to find feasible first-stage decisions for different scenarios and maximise the difference between the optimum solution for each scenario and robust solution. The authors propose a new robust optimisation method for generic logistics management problem and compare the results and computational times with the second approach which are found by Mulvey and Ruszczyński (1995), Mulvey and Vanderbei (1995) and Yu and Li (2000).

The previous work on the planning of disassembly, refurbishing and remanufacturing in closed loop supply chains is concentrated on the development of deterministic models with details of multi-echelon nature of the network or stochastic models that focused on one or two echelons of the complex network. In this paper, we consider a more general closed loop manufacturing system as a whole with multiple modular products and develop stochastic and robust optimisation models that reflect all the manufacturing decisions at the strategic and tactical levels representing the realistic nature of the complex decisions in this system. We present two different solutions approaches, SOA and ROA, to analyse the effect of uncertainties in the system. We consider a two-stage optimisation problem such that in the first stage, we focus on the capacity decisions for the facilities in this system and in the second stage, we consider a multi-period remanufacturing problem with given production capacities and focus on the optimal production, disposal and inventory decisions using a large-scale mixed integer linear programme. In this paper, Section 2 builds a generic disassembly and remanufacturing model which can be used in the problems with product modularity such as personal computers, mobile phones, automobiles, etc. and present how SOA and ROA are implemented in the generic model. Then, Section 3 shows the experimental design of the problem and show the results of deterministic, stochastic and robust optimisation approaches. Finally, Section 4 concludes the study.

2. Problem statement and model

In this paper, we model collection, disassembly, polishing and refurbishing operations of products while we minimise the total operating, purchasing and holding costs. We consider a multi-period production system that collects used products from end users, inspects them and then either disposes or uses them in production depending on the condition of the product and the needs of the system to obtain the highest benefit from these products. At every period, a random amount of used products are collected and a random amount of demand for the final products are observed. The collected used products are disassembled into their parts and these parts are reused in the production of new products after being refurbished. We assume that the final products can either be manufactured by using the parts obtained from the used products or by using virgin parts bought from an external supplier, which is considered to be more costly than employing refurbished parts.

The schematic representation of disassembly and refurbishing operations of a generic remanufacturing case is shown in Figure 1. The solid arrows denote products and the dashed arrows denote parts. A typical real-life example for this generic representation includes electronics and computer manufacturing industries. Especially, portable electronic and communication devices including cellular and smart phones, tablet computers are becoming a major concern due to their short life cycles and the valuable parts and materials contained in these devices. In these industries, many parts and components that pass certain quality specifications and standards can be reused in refurbished or remanufactured products in addition to new products. If some parts do not pass the quality specifications or thresholds, then they are disposed off in landfills or subjected to major recycling operations to extract valuable materials in them.

Collected products are held in the collection site when they arrive to the system and there is a certain holding cost to store these products. We consider different types of products, each consisting of different parts and requiring different types of raw materials in production. The collected products can also be of various types and can be in different conditions and at different quality levels. Some of these products can be sold as second-hand products after a small operation. Such products are used to fulfill the possible demand for the second-hand products. Some of the collected products might be in a very bad condition and thus should be sent to the disposal site as waste. These products are sent to the landfill sites and thus has a certain cost to the company. Other products can be repaired or refurbished depending on their quality level and type, and then used in the production of new products. These products are sent to the disassembly site and they are disassembled into their parts incurring a certain operating cost. As mentioned above, each product type includes different types of parts and materials. After the disassembly operation, some parts can be used in the remanufacturing operation and thus are sent to the refurbishing site, while others might be beyond repair or reuse and thus needs to be sent to the disposal site. The useful parts are refurbished in the refurbishing site and then used as raw materials in the manufacturing process.

After refurbishing, parts are held in the refurbishing site or part inventory site and they are used to satisfy the raw material demand in the manufacturing facility in the future periods. Since, it might not be possible to satisfy all the raw material demand from used products, we consider an outside supplier such that the manufacturer can buy the required raw materials as new from this supplier at a certain cost, which is possibly higher than the cost of obtaining these materials from used products. We also assume that if the amount of the raw materials obtained from used products exceeds the needs in the production process, the excess amount can be sold to third parties at a salvage price, providing a revenue to the company. The salvage price of each excess raw material is considered to be lower than the purchasing price of these materials from the outside supplier, since otherwise the system will salvage all of these parts and buy all the required raw materials from the outside supplier. Thus, in our model, the system will only salvage the parts obtained from used products if there is an excess amount after satisfying the raw material demand in the manufacturing process.

In this multi-period production planning system, the capacities of the disassembly and refurbishing facilities play an important role and we determine the optimal capacities of these sites as strategic-level decisions for the company considering

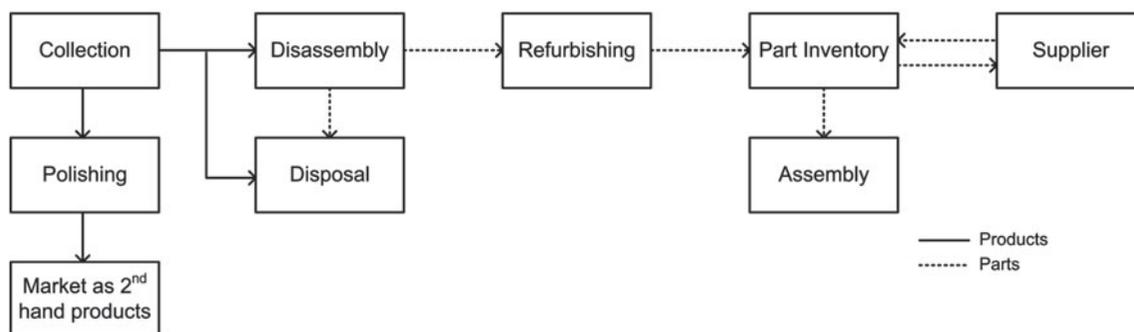


Figure 1. Schematic representation of the remanufacturing system.

the uncertainties in the system in the future periods. We note that these strategic decisions should be given at the beginning of the planning horizon without observing the collected amounts of used products and the demand for the final products in the future periods and it is not easy to change the capacities of manufacturing facilities in a short time after the operations have started. We consider these decisions as the first-stage decisions in our two-stage stochastic optimisation procedure and we aim to determine the optimal capacities in this system. Then, in the second stage of our model, we determine the optimal disposal, production and inventory values at each period after observing the return and demand values.

We present the second-stage large-scale mixed integer linear deterministic programme with its explanations in Appendix 1, assuming that the capacity decisions are given. In the following sections, considering the uncertainties in the system, we focus on the first-stage decisions about the capacity levels in this system and use the model A as the second phase of our two-stage optimisation problem.

2.1 Two-stage stochastic programming approach

In optimisation problems, decisions may have different priority levels due to the importance, cost or time restriction of these decisions. In our remanufacturing problem, we decompose our mathematical programming model into two stages considering long term and short term decisions. In stochastic programming approach, we consider these two stages as strategic- and tactical-level decisions. Strategic-level decisions are the first-stage decisions and tactical-level decisions are the second-stage decisions. We use a two-stage stochastic programming approach to handle uncertainty in demand and supply quantities. Strategic-level decisions should be made before the system starts. Thus, first-stage decisions should be implemented before the actual demand and supply quantities are known. After the demand and supply quantities are observed, second-stage decisions are given according to these values.

The capacities of disassembly and refurbishing sites are the first-stage decisions in our optimisation problem. The number of disassembly sites, Dse , and the number of refurbishing sites, Rfe , are the first-stage integer decision variables. These decision variables can be considered as the number of machines to be bought or number of workers to be employed in the system. We determine different scenarios to handle uncertainty in demands and returns for parts and products, respectively. We use S as the set of demand-supply scenarios and s as the specific scenario in set S .

In order to handle uncertainty of demand and supply, we use the deterministic model A in the following stochastic optimisation model. The strategic-level decisions that represent the capacity of a facility, Dse and Rfe , are shown as the vector y and other variables used in the second-stage problem are shown as the vector x in the two-stage stochastic programming approach. Thus, variables y are used as first-stage variables and variables x are used as second-stage variables. The two-stage stochastic programming model for our problem is shown as below in a compact form considering the model in Appendix 1 with different possible scenarios.

$$\text{Min } fy + E_s [Q(y, s)] \quad (2.1)$$

s.t.

$$y \in Z^+ \quad (2.2)$$

where

$$Q(y, s) = \min(cx) \quad (2.3)$$

s.t.

$$W_1x = b(s) \quad (2.4)$$

$$W_2x = d(s) \quad (2.5)$$

$$W_3x \leq Ty \quad (2.6)$$

$$x \in Z^+ \quad (2.7)$$

Equation (2.1) is the objective function of the first stage in the two-stage stochastic optimisation approach. First stage y variables are determined in this stage and expectation of $Q(y, s)$ over s plus fy gives the total cost of the optimisation problem. Here $Q(y, s)$ represents the minimum cost solution for scenario s while f is the unit cost vector of the capacity decisions and y includes the capacity decision variables in the first stage. Observe that second-stage decisions x are given considering different scenarios and their expected cost values are considered in the first stage to decide on the y variables. Equation (2.3) shows the second-stage objective function where $Q(y, s)$ denotes the minimum value for the objective function

for a particular scenario s . Parameters $b(s)$ and $d(s)$ are used for supply and demand quantities for particular scenario s , respectively and W is used as resource matrices. Equations (2.4) and (2.5) are used to satisfy demand and supply restrictions where each constraint depends on scenario s . Equation (2.6) is used to show scenario-independent constraints.

This two-stage stochastic optimisation method creates different sets of variables for each particular scenario s and we show them as x_s . In this approach, which is originally proposed by Dantzig (1955), probabilities for each scenario are assigned and a solution for each scenario is obtained (see Birge and Louveaux 1997; Kall and Wallace 1994). However, the optimal solution for particular scenario s is not optimal for the general problem. Consequently, the above two-stage problem is formulated as follows such that it can be solved by commercial solvers as a single-stage optimisation problem to find the optimal values for these variables considering different scenarios. However, note that with this stochastic programming approach, the problem size increases with the number of scenarios considered.

$$\text{Min } fy + E_s [cx_s] \quad (2.8)$$

s.t.

$$W_1x_s = b(s) \quad \forall s \in S \quad (2.9)$$

$$W_2x_s = d(s) \quad \forall s \in S \quad (2.10)$$

$$W_3x_s \leq Ty \quad \forall s \in S \quad (2.11)$$

$$y \in Z^+ \quad (2.12)$$

$$x_s \in Z^+ \quad \forall s \in S \quad (2.13)$$

In our formulation the list of decision variables, y , in the first stage includes the strategic decisions in the model A related to the capacities of the facilities, Dse and Rfe and the array of decision variables in the second stage, x_s , includes the operational variables in the model A such as inventory levels, amount of products and parts processed at each stage in every period for each scenario s .

2.2 Robust optimisation approach

Although stochastic programming approach minimises the expected cost over all scenarios, robust optimisation aims to decrease the variability of scenario costs from the optimal solution, as stated in Kouvelis and Yu (1997). Robust optimisation model aims to find a solution that minimises the maximum deviation of the resulting costs from the optimal solution over all possible scenarios as follows:

$$\text{Min } \delta \quad (2.14)$$

s.t.

$$\delta \geq R_s - Q_s^* \quad \forall s \in S \quad (2.15)$$

$$W_1x_s = b(s) \quad \forall s \in S \quad (2.16)$$

$$W_2x_s = d(s) \quad \forall s \in S \quad (2.17)$$

$$W_3x_s \leq Ty \quad \forall s \in S \quad (2.18)$$

$$y \in Z^+ \quad (2.19)$$

$$x_s \in Z^+ \quad \forall s \in S \quad (2.20)$$

where

Q_s^* : Cost of optimal solution for scenario s

R_s : Cost of robust solution for scenario s

As mentioned in the stochastic programming approach, our aim in the robust optimisation approach is to determine the long-term capacity decisions first and then we determine the short-term operational decisions for given capacity levels. We connote S as the set of demand-supply scenarios and s as the specific scenario as in the stochastic programming approach. We do not assign any probabilities in the robust programming model since we do not take an expectation and we aim to minimise the maximum deviations over all scenarios. First, we solve all scenarios separately and find the optimal cost value for each scenario, denoted as Q_s^* above for scenario s . In the above model, R_s denotes the robust solution cost for each scenario s . Then, using Equation (2.14) and Constraint (2.15), using δ , we minimise the difference between the optimum cost and the robust cost considering all scenarios. Equations (2.16)–(2.20) are the same as in the stochastic programming constraints. Thus, y variables are used for strategic-level decision variables and they are given in the first stage of the problem. Then, x values are determined as tactical-level decisions. The same variables are included in variable set y and x in stochastic and robust optimisation problems. These two approaches are implemented in GAMS optimisation environment (see Rosenthal 2012).

3. Numerical results

In this section, we present the numerical results of our study to analyse the effects of uncertainty on the system results and to compare the performances of the stochastic optimisation and the robust optimisation approaches in this system.

3.1 Input Parameters

In our computational study, we consider a planning horizon of 20 periods and assume that 5 different types of parts are used to produce three different types of final products. The prices of the parts (PuC) are 200, 150, 50, 100 and 150 for parts 1, 2, 3, 4 and 5, respectively, if they are purchased as new from the suppliers. The holding cost for each part per period is considered to be 1% of that part's purchase cost before the refurbishing activity and 2% of the part's purchasing cost after that part is refurbished and moved to the part inventory site. The refurbishing cost for a part (ORF) is taken as 40% of the purchasing cost of that part and the salvage price of a part is considered to be 50% of the purchasing cost of a new part. The volume for one unit part i (PaV), the disposal cost for part i ($PaDC$), the upper bound of disposal rate for part i (PaU), the necessary time for refurbishing one unit of part i (PaT), and initial inventories for part inventory site ($xIPA$) and refurbishing sites ($xIRF$) for each part i are shown in Table 1.

The remanufactured final products in the system are sold as remanufactured products at a price of ($PrSe$) 200 for all product types. The unit operation cost for product p at the disassembly site (OD) and polishing site (OPO) are 20% of the selling prices of that product. The unit inventory holding cost for product p is taken to be 2% of the product's selling price. In addition to this, the capacity of collection site ($PrIC$), the disposal cost ($PrDC$), the upper bound of disposal rate (PrU), the necessary time to disassemble one unit of product (PT), initial inventories at collection site ($xIPR$) and initial inventory at second-hand products site ($xISH$) are shown in Table 2.

The bill of materials for each product p is shown in Table 3. For example product type 1 requires one unit of part types 1–3.

Table 1. Data-set for part types.

	PaV	PaDC	PaU	PaI	PaIRF	ORF	PuC	PaSe	PaT	xIPA	xIRF
Pa1	1	2	0.2	4	2	80	200	100	1	0	0
Pa2	1	1.5	0.2	2.5	1.5	60	150	75	1	0	0
Pa3	1	0.5	0.2	1	0.5	20	50	25	1	0	0
Pa4	1	1	0.2	2	1	40	100	50	1	0	0
Pa5	1	1.5	0.2	2.5	1.5	60	150	75	1	0	0

Table 2. Data set for product types.

	PrIC	PrDC	PrU	PrI	PrIS	OD	OPO	PrSe	PrT	xIPR	xISH
Pr1	1,000	2	0.3	40	4	20	20	200	2	0	0
Pr2	1,000	2	0.3	40	4	20	20	200	2	0	0
Pr3	1,000	2	0.3	40	4	20	20	200	2	0	0

Table 3. The quantity of part i from product p .

	$BM_{p,i}$				
	Pa1	Pa2	Pa3	Pa4	Pa5
Pr1	1	1	0	1	0
Pr2	1	0	1	0	1
Pr3	0	1	1	1	0

Table 4. Scalars

PaIC	OC	RFC	DSC	Dsec	Rfec
400	10,000	100	100	100,000	20,000

Table 5. Distributions of scenarios.

		Return		
		Low $\mathcal{N}(30, 9)$	Medium $\mathcal{N}(120, 36)$	High $\mathcal{N}(240, 72)$
Demand	Low $\mathcal{N}(50, 15)$	Scenario 1	Scenario 4	Scenario 7
	Medium $\mathcal{N}(200, 60)$	Scenario 2	Scenario 5	Scenario 8
	High $\mathcal{N}(400, 120)$	Scenario 3	Scenario 6	Scenario 9

In addition to parameters for different types of products or parts, we have 6 more different scalars. The volume capacity of part inventory site at each period ($PaIC$), the ordering cost from supplier (OC), the time capacity of refurbishing site at each period (RFC), the time capacity of disassembly site (DSC), the opening cost of disassembly site ($Dsec$) and the opening cost of refurbishing site ($Rfec$) are shown in Table 4.

Assuming that the demand for the final products and the return amounts of the used products can be low, medium or high, we consider 9 different scenarios in our numerical calculations as shown in Table 5. In each scenario, we assume independent normal distributions with different means and variances for the demand (D) and supply (PrC) amounts in each period. We assume that 60% of the sold products are returned at all cases, thus, the mean levels for the returns are taken as 60% of the mean levels for demand. In addition, we assume that the standard deviations of the distributions in each scenario is 30% of the mean values. For example, in scenario 1 both demand and supply have a low level of normal distribution, thus, each type of part demand at each period are taken from a normal distribution with mean 50 and standard deviation 15 and returns come to the system according to a normal distribution with mean 30 and standard deviation 9. We note that the negative realisations are truncated to 0. In addition to part demand and product return, second-hand product demand is also random and the demand for second-hand products (DS) is assumed to follow a normal distribution $\mathcal{N}(20, 6)$ in all scenarios.

We used GAMS 23.3.3 and Cplex 12.1.0 for solving the proposed models on a PC with Intel Xeon 3.0 GHz and 4 GB of Ram. We used the standard branch-and-bound algorithm of the CPLEX solver for stochastic and robust optimisation problems using the same settings. Therefore, the comparison of the quality of the solutions for stochastic and robust optimisation approaches would be consistent.

3.2 Comparison of the models

In this section, we present the results of our analysis considering the data as explained in the previous section. In Table 6, we present the optimal first stage decisions for each scenario assuming that the demand and return levels are known to behave according to that scenario beforehand. When we analyse Table 6, we see that the company opens only one disassembly site (Dse) and 2 refurbishing sites (Rfe) on average when return levels are low. Even when the demand level is high, the optimal values of Rfe and Dse do not change since there are not enough returns to process. However, when return level becomes

Table 6. Optimum first stage decisions for each scenario and by SOA and ROA.

Scenario	Optimum Rfe	Optimum Dse
1	2	1
2	2	1
3	2	1
4	4	3
5	7	5
6	8	5
7	5	5
8	12	9
9	20	13
SOA	9	6
ROA	10	7

Table 7. The lower bound of the costs and the costs with SOA and ROA for each realised scenario.

Scenario	Lower bound	SOA	ROA
1	592,612	1,154,657	1,274,657
2	2,727,798	3,332,050	3,452,050
3	6,150,498	6,752,290	6,872,290
4	574,640	900,548	1,119,854
5	2,329,224	2,382,808	2,566,746
6	5,872,264	5,882,430	5,946,658
7	2,424,865	2,567,398	2,666,283
8	2,289,123	2,722,146	2,472,669
9	6,003,770	7,024,522	6,732,861
Expected cost	3,218,310	3,635,427	3,856,007
Max. loss		1,020,752	729,091

medium or high, additional facilities are opened and the number of these facilities increase with the demand level. We also observe that the return level has a higher impact than the demand level on the optimal decisions.

Note that even though each scenario has its own optimum solution, we need to decide on the optimal first-stage decisions without having any information about which scenario will be realised. Thus, using the approaches as explained in Section 2 we solve the problem using the two-stage stochastic and robust optimisation approaches, by generating a representative sample realisations of demand and return values obtained from each scenario with equal probabilities. We determine the optimal decisions and calculate the expected costs for both *SOA* and *ROA*. In Table 6, we present the first-stage decisions for these two different approaches and observe that the optimal first-stage decisions are different than the optimal solutions in any of the scenarios.

In Table 7, we present the average cost values of the sample realisations of each scenario, when the first-stage decisions are given according to *SOA* or *ROA* but the actual demand and return values are realised according to that scenario. The first column in the table provides the average of the lower bounds on the optimal costs, which are obtained by assuming that the actual realisation of demand and return values are known beforehand and the first-stage decisions are set deterministically according to that sample realisation of that scenario. We observe that the costs with *SOA* is about 13% higher than the lower bound, that is the uncertainty in the system causes an efficiency loss of about 13% on average. The expected costs increase even further when *ROA* is used and reaches to a level that is about 19% higher than the lower bound. However, when we look at the worst-case scenarios, we observe that the maximum difference of the costs that can be realised with respect to the lower bound can reach to a level of 1,020,752 when *SOA* is used, while the maximum difference with *ROA* is 729,091 which is about 30% less than the efficiency loss with *SOA*. Thus, as expected even though *SOA* performs better on average, *ROA* results in a lower efficiency loss in the worst case.

Table 8. Sensitivity analysis over the product holding cost.

PrI	Avg. lower bound	SOA				ROA			
		Rfe	Dse	Average cost	Max. loss	Rfe	Dse	Average cost	Max. loss
0	2,865,124	3	2	3,090,465	431,157	3	2	3,090,465	346,253
1	2,984,513	5	3	3,307,923	646,782	6	3	3,354,746	489,642
2	3,104,365	6	4	3,482,759	811,034	7	4	3,693,654	603,416
3	3,192,759	7	5	3,593,812	932,536	8	6	3,803,528	676,784
4	3,218,310	9	6	3,635,427	1,020,752	10	7	3,856,007	729,091
5	3,221,124	9	6	3,641,856	1,023,465	10	7	3,863,231	730,120
6	3,222,135	9	6	3,642,914	1,024,573	10	7	3,864,465	730,542

Table 9. The comparison of demand vs. return variability.

	Avg. lower bound	SOA				ROA			
		Rfe	Dse	Average cost	Max. diff.	Rfe	Dse	Average cost	Max. diff.
Variable return	2,517,433	11	8	2,936,778	862,615	9	7	3,147,028	722,615
Variable demand	2,769,680	6	4	2,836,370	129,049	6	4	2,836,370	129,049
Medium return and demand	2,371,192	8	5	2,374,399	18,328	8	5	2,374,399	18,328
Variable return and demand	3,218,310	9	6	3,635,427	1,020,752	10	7	3,856,007	729,091

3.3 Sensitivity analysis for the inventory holding cost

In this section, we aim to analyse the importance of the inventory holding costs in the system and thus do a sensitivity analysis of the system by varying PrI . In Table 8, we present the optimal first-stage decisions and the resulting cost values with SOA and ROA as well as the lower bounds for the average costs and the worst-case performances of SOA and ROA for varying PrI values. We observe that more facilities are opened as the inventory holding cost increases. In addition, average cost values increase in a concave manner as the inventory holding cost increases and the cost values reach to a steady level when the inventory holding cost gets large enough. This is due to the fact that, after a certain level, the model chooses to open more facilities to increase the capacity instead of carrying inventories and thus keeps very low levels of inventory and increasing the holding cost even further will not effect the solution much. We also observe that both SOA and ROA perform better when the inventory holding cost is low and the cost of uncertainty increases with PrI . When holding cost is low, the uncertainty in the system can be handled with lower costs through inventories, however, as inventory costs increase, the randomness becomes more costly for the system.

3.4 Comparison of demand vs. return variability

In this section, we aim to analyse the importance of the randomness of demand and return values and identify the importance of each of them for the system. For this purpose, first we assume that the demand is known to be distributed according to its medium level while the distribution of the returns can be low, medium or high as given in the previous sections. The results for this case are presented in the first row of Table 9. Then we assume that the amount of returns are distributed according to its medium level while the demand distribution can be one of its low, medium or high levels and the results are presented in the second row of Table 9. Finally, we assume that both the demand and return amounts are distributed according to their medium levels and we present the results in the third row of Table 9. We also present the results when both demand and returns can be distributed according to their low, medium or high levels in the last row of Table 9.

We observe that the variability of return is the main issue in the system and is much more important than the randomness of demand since SOA results in about 17% more costs than the lower bound when the returns are variable while this difference is only about 2% when the return level is fixed and the demand is the only source of randomness. We also observe that SOA and ROA both result in similar performances which are very close to the lower bound when the return levels are fixed. However, when return levels can vary, the expected costs with ROA becomes about 25% more than the lower bound, which appears due to the randomness of the return levels. When we consider the worst case performances, we see a similar result

such that the maximum difference from the lower bound is only 129,049 when only demand is random. However, when return level is random, this difference increases to 722,615 with *ROA* and to 862,615 with *SOA*. Finally, we observe that, when the distribution of both the demand and return levels are known, the variability of these values across periods has very little effect on the system. In this case, *SOA* and *ROA* give the same results and perform very close to the deterministic case.

4. Conclusion

In this paper, considering the uncertainties in the system, we develop a large-scale mixed integer mathematical optimisation model for the capacity planning, production and inventory decisions in a closed loop manufacturing system for modular products. We develop both a stochastic optimisation and a robust optimisation model in which there are two stages, considered as strategic and tactical levels. In the first stage, considering the uncertainty about the demand and return levels and their effects on the second-stage operations, the model decides on the capacity of the system by determining the optimal number of disassembly and refurbishing sites to open. Then, in the second stage, after observing the actual demand and return values, operational decisions, such as the production, disposal and inventory rates are given for fixed capacities.

After developing the model for this remanufacturing problem, we make an extensive numerical study to analyse the system behaviour. We observe that the uncertainty in the system causes an efficiency loss of about 13% on average when the stochastic optimisation approach is used. The efficiency loss increases even further to 19% when robust optimisation is used. When we consider the worst-case performances, we observe that stochastic optimisation performs about 30% worse than robust optimisation. We also analyse the effect of uncertainties in demand and return amounts and we observe that the variability in return is much more important than the variability of demand in this system. The distribution of the collected amounts of used products have the highest impact in the capacity decisions since these amounts are the ones that need to be processed in the system and they effect the decisions of the model much more than the distribution of demand. Thus, focusing on correctly estimating the amount of returns is much more important for the managers of this remanufacturing system than correctly estimating the demand.

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Appendix 1. The mathematical model

Sets:

- Set of parts I ; index $i \in I$
- Set of products P ; index $p \in P$
- Set of periods T ; index $t \in T$

Decision variables:

- IPR_{pt} : the inventory level for product p at time t at collection site
- ISH_{pt} : the inventory level for product p at time t at 2nd hand products site
- IPA_{it} : the inventory level for part i at time t at part inventory site
- IRF_{it} : the inventory level for part i at time t at refurbishing site
- $PrDA_{pt}$: the quantity of disassembled product p at time t
- $PrDS_{pt}$: the quantity of product p sent to the disposal site at time t
- $PrPO_{pt}$: the quantity of polished product p at time t
- $PrSH_{pt}$: the quantity of second-hand product p sold at time t
- $PaDS_{it}$: the quantity of part i sent to the disposal site at time t
- $PaDA_{it}$: the quantity of part i sent to the refurbishing site at time t
- $PaRF_{it}$: the quantity of refurbished part i at time t
- $PaSUp_{it}$: the quantity of sold part i to the supplier at time t
- PaB_{it} : the quantity of purchased part i at time t from supplier
- Ord_{it} : the binary variable for ordering from supplier at time t
- Dse : the number of disassembly site
- Rfe : the number of refurbishing site

Parameters:

- RFC : the time capacity of refurbishing site at each period
- DSC : the time capacity of disassembly site at each period
- PrC_{pt} : the quantity of collected product p at time t
- D_{it} : the demand for part i at time t
- PaV_i : the volume for one unit part i
- $PrIC_p$: the capacity of collection site for each product p
- $PaIC$: the volume capacity of part inventory site at each period
- DS_{pt} : the demand of second-hand product p at time t
- BM_{pi} : the quantity of part i from product p
- $PrDC_p$: the disposal cost for product p
- $PaDC_i$: the disposal cost for part i
- PrU_p : the upper bound of disposal rate for product p
- PaU_i : the upper bound of disposal rate for part i
- PrI_p : the unit inventory holding cost for product p at collection site
- PaI_i : the unit inventory holding cost for part i at the part inventory
- $PaIRF_i$: the unit inventory holding cost for part i at refurbishing site
- $PrIS_p$: the unit inventory holding cost for product p at second-hand site

OD_p : the unit operating cost for product p at the disassembly site
 ORF_i : the unit operating cost for part i at the refurbishing site
 OPO_p : the unit operating cost for product p at the polishing site
 OC : the order cost from supplier
 $PuCi$: the purchasing cost for part i
 $PaSe_i$: the selling price of part i
 $PrSe_p$: the selling price of second-hand product p
 PrT_p : the time needed for disassembling one unit of product p
 PaT_i : the time needed for refurbishing one unit of part i
 $Dsec$: the opening cost of disassembly site
 $Rfec$: the opening cost of refurbishing site

Objective Function:

$$\text{Min}z = \sum_p \sum_t PrI_p * IPR_{pt} \quad (\text{A.1})$$

$$+ \sum_p \sum_t PrIS_p * ISH_{pt} \quad (\text{A.2})$$

$$+ \sum_p \sum_t PrPO_{pt} * OPO_p \quad (\text{A.3})$$

$$+ \sum_p \sum_t PrDA_{pt} * OD_p \quad (\text{A.4})$$

$$+ \sum_p \sum_t PrDS_{pt} * PrDC_p \quad (\text{A.5})$$

$$+ \sum_i \sum_t PaRF_{it} * ORF_i \quad (\text{A.6})$$

$$+ \sum_i \sum_t PaI_i * IPA_{it} \quad (\text{A.7})$$

$$+ \sum_i \sum_t PaIRF_i * IRF_{it} \quad (\text{A.8})$$

$$+ \sum_i \sum_t PaDS_{it} * PaDC_i \quad (\text{A.9})$$

$$+ \sum_i \sum_t PaB_{it} * PuC_i \quad (\text{A.10})$$

$$+ \sum_t Ordrt * OC \quad (\text{A.11})$$

$$+ (Dse) * Dsec \quad (\text{A.12})$$

$$+ (Rfe) * Rfec \quad (\text{A.13})$$

$$- \sum_i \sum_t PaSUp_{it} * PaSe_i \quad (\text{A.14})$$

$$- \sum_p \sum_t PrSH_{pt} * PrSe_p \quad (\text{A.15})$$

s.t.

$$PrC_{pt} + IPR_{p(t-1)} = IPR_{pt} + PrDA_{pt} + PrDS_{pt} + PrPO_{pt} \quad \forall p, t \quad (\text{A.16})$$

$$\sum_p BM_{pi} * PrDA_{pt} = PaDS_{it} + PaDA_{it} \quad \forall i, t \quad (\text{A.17})$$

$$\sum_t PrDS_{pt} \leq PrU_p * \sum_t PrC_{pt} \quad \forall p \quad (\text{A.18})$$

$$\sum_t PaDS_{it} \leq PaU_i * \sum_t (PaDS_{it} + PaDA_{it}) \quad \forall i \quad (\text{A.19})$$

$$PrDS_{pt} + PrPO_{pt} + PrDA_{pt} \leq PrIC_p \quad \forall p, t \quad (\text{A.20})$$

$$\sum_p PrT_p * PrDA_{pt} \leq Dse * DSC \quad \forall t \quad (A.21)$$

$$\sum_i PaT_i * PaRF_{it} \leq Rfe * RFC \quad \forall t \quad (A.22)$$

$$IRF_{i(t-1)} + PaDA_{it} = IRF_{it} + PaRF_{it} \quad \forall i, t \quad (A.23)$$

$$ISH_{p(t-1)} + PrPO_{pt} = ISH_{pt} + PrSH_{pt} \quad \forall p, t \quad (A.24)$$

$$IPA_{i(t-1)} + PaRF_{it} + PaB_{it} = D_{it} + PaSUP_{it} + IPA_{it} \quad \forall i, t \quad (A.25)$$

$$\sum_i PrV_i * IPA_{it} \leq PaIC \quad \forall t \quad (A.26)$$

$$PaB_{it} \leq D_{it} * Ordr_t \quad \forall i, t \quad (A.27)$$

$$PrSH_{pt} \leq DS_{pt} \quad \forall p, t \quad (A.28)$$

$$PaSUP_{it} \leq D_{it} \quad \forall i, t \quad (A.29)$$

$$Dse \geq 1 \quad (A.30)$$

$$Rfe \geq 1 \quad (A.31)$$

$$IPR_{pt}, ISH_{pt}, PrDA_{pt}, PrDS_{pt}, Dse, Rfe \in \mathbb{Z}^+ \quad \forall p, t \quad (A.32)$$

$$IPA_{it}, IRF_{it}, PaDS_{it}, PaSUP_{it} \in \mathbb{Z}^+ \quad \forall i, t \quad (A.33)$$

$$PrPO_{pt}, PrSH_{pt} \geq 0 \quad \forall p, t \quad (A.34)$$

$$PaRF_{it}, PaB_{it}, PaDA_{it} \geq 0 \quad \forall i, t \quad (A.35)$$

$$Ordr_t \in \{0, 1\} \quad \forall t \quad (A.36)$$

The objective of the optimisation model is to minimise the total cost minus the total profit (Equations (A.1)–(A.15)). The first 13 terms of the objective function give the cost of operations while the revenues generated by selling parts (Equation (A.14)) and second-hand products (Equation (A.15)) are given by the last two terms. The constraints of the problem include inventory balance, parts balance, capacity restrictions and supply and demand constraints. The inventory balance at the collection site for each part is given by Equation (A.16). The parts balance at the disassembly site is given by Equation (A.17) and the restrictions on the disposal quantities are enforced in Equations (A.18) and (A.19). The capacity restriction of disassembly site is given by Equation (A.20). The expansion of disassembly and refurbishing sites are modelled with Equation (A.21) and Equation (A.22), respectively. The inventory balance for the refurbishing site, second-hand site and the part inventory facility are given by Equations (A.23)–(A.25) respectively. The storage limit of parts for each time period t of operation is given by Equation (A.26). The restriction on the purchase amount of parts for each period t is given by Equation (A.27). The sale of second-hand products cannot exceed the demand for these products as given by Equation (A.28). The restriction on the sale of recovered parts to parts supplier are given by Equation (A.29). The closed loop supply network includes at least one disassembly and one refurbishing site as given by Equation (A.30) and Equation (A.31), respectively. Equations (A.32)–(A.36) declares the type of the decision variables.